MathLab: MAGIC SQUARES

Numbers and their relationships to the real world and to each other have mystified civilizations for thousands of years. The ancient Chinese were no strangers to mathematics and sometimes combined their mathematics with mysticism. One of their legacies is that of magic squares. The oldest known example of a magic square is the *lo-shu*, which myth claims was first seen by Emperor Yu about 2200 B.C. decorated on the back of a divine tortoise along the bank of the Yellow River (See what I mean about mysticism?) A MAGIC SQUARE is a square array of integers so arranged that the different numbers along any row, column, or main diagonal have the same sum, called the **magic constant** of the square. A magic square is said to be normal if the $n^2$ numbers are the first $n^2$ positive integers. For instance in a 3 by 3 magic square the integers 1 through 9 are used. In a 5 by 5 magic square only the integers 1 through 25 are used. The magic constant of an nth order normal magic square can be found using a formula. The formula is

$$\text{magic constant} = \frac{1}{2} n (n^2 + 1)$$

So for a 3rd order (3 by 3) magic square the magic constant would be

$$\frac{3}{2} \left( 3^2 + 1 \right) = \frac{3 \cdot (10)}{2} = \frac{30}{2} = 15$$

Show that the magic constant for a 5th order (5 by 5) magic square would be 65.

What is the magic constant of a 9th order magic square?

In the 3 by 3 array below try to arrange the integers 1 through 9 so that each row, column, and main diagonal have the sum 15. Take a few minutes in the attempt. Many different such squares are possible.
It probably took you at least a little time to fit the integers in so that each row, column, and diagonal had a sum of 15. Think how much longer it would take if we were constructing a magic square of order 5 or 7 or 9. Fortunately there is an easier way than trial and error. In 1687-1688 a man named De la Loubere learned a simple method for finding a normal magic square of any odd order. We will illustrate the method by constructing one of the 5th order. We start by bordering the 5 by 5 square array below with cells along the top and right edge, and shading the added cell in the top right corner.
Write 1 in the middle top cell of the original square. The general rule is then to proceed diagonally upward to the right with successive integers. Exceptions to this general rule occur when such an operation takes us out of the original square or leads us into a cell already occupied. In the first situation we get back into the original square by shifting clear across the square, either from top to bottom or from right to left, as the case may be, and continue with the general rule. In the second situation we write the number in the cell immediately beneath the one last filled, and then continue with the general rule. The shaded cell is to be regarded as occupied.

Thus, our general rule would have us place a 2 diagonally upward in the fourth cell bordered along the top. We must, therefore, shift the 2 to the fourth cell in the bottom row of the original square (see below). Continuing with the general rule would have us put 4 in the third cell up bordered along the right edge. It must, therefore, be written clear across to the left in the third cell up in the first column of the original square.

The general rule would place 6 in the cell already occupied by 1. It is accordingly written in the cell just below that occupied by the last written number, 5 (see below).
See if you can finish the magic square and check to see if your rows, columns, and main diagonals have the sum of 65. Check with your teacher if you run into problems but try to work them out in your groups.

If you think you have mastered the technique try constructing a magic square of order 7. What is the magic constant of such a square?
There is an easy way to construct magic squares of doubly-even order, that is, magic squares whose orders are multiples of four. Consider, first of all, a square of order four and visualize the diagonals as drawn (see figure below).

Beginning in the upper left corner, count across the rows from left to right in descending succession, recording only the numbers in cells not cut by a diagonal. Now, beginning at the lower right corner, count across the rows from right to left in ascending succession, recording only the numbers in cells that are cut by a diagonal. The same rule applies to any magic square of order $4n$ if we visualize the diagonals of all the $n^2$ principal four-by-four sub-blocks.

Complete the 8 by 8 magic square below. What is the magic constant for this magic square? See if you can find any other magic properties of this magic square.
8×8 Magic Square

With main and 4×4 diagonals