1.3 Constructing Parallel and Perpendicular Lines

Parallel lines are coplanar lines which do not intersect. **Perpendicular lines** are two lines that intersect to form right angles. The next three constructions involve constructing parallel and perpendicular lines.
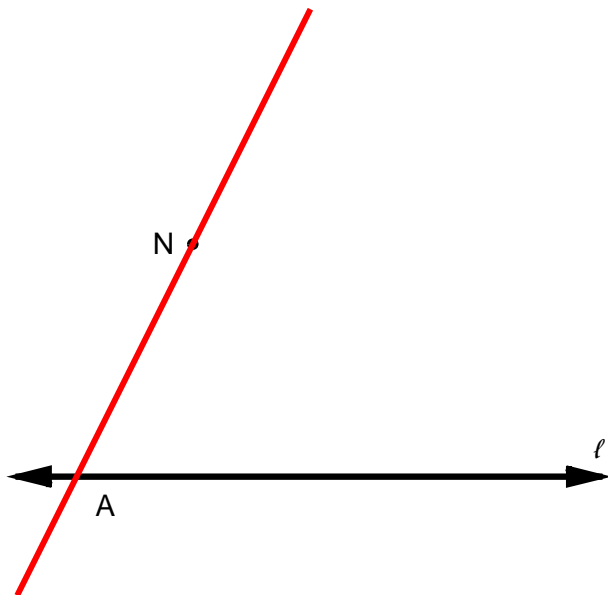
Construction 5: Parallel through a Point Not on a Line

In this exercise we will construct a line parallel to a given line through a specific point not on the line. First start with a figure similar to the one below.
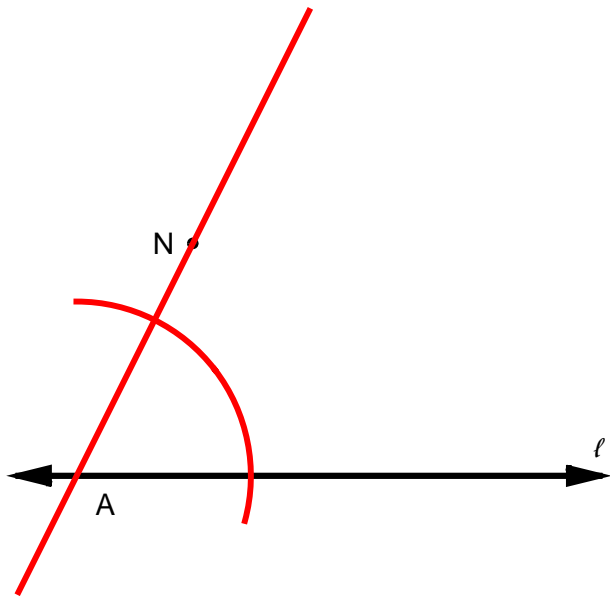
N •

ℓ

A

Draw a line through N that intersects line l. Label the point of intersection A. You should have something like this:

N •

ℓ

A

Now we need to construct an angle with vertex N congruent to the angle at vertex A. This involves using construction #2. First put the pointed end of the compass on point A and draw an arc that intersects both sides of the angle as illustrated below.

Keeping the compass setting the same make a similar arc with the compass point on point N as illustrated below.
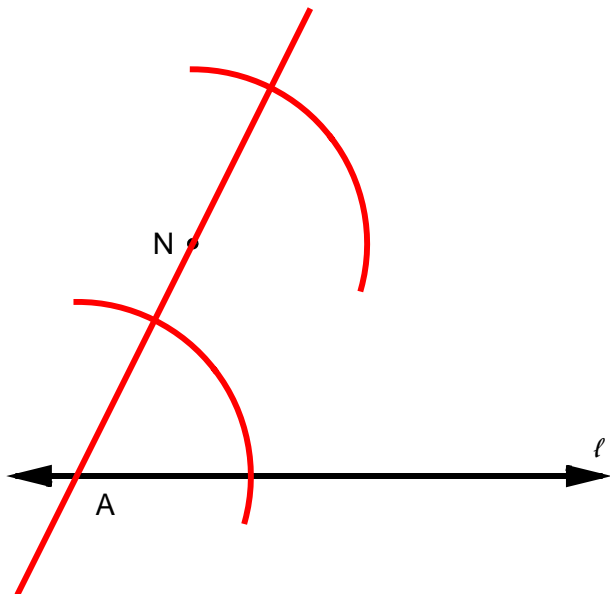

Continue as in construction #2. Place the pointed end of the compass on the point where the first arc you drew intersects $\overleftrightarrow{NA}$. Set the compass to the distance between that point and the point where the arc intersects line l. Make a small arc at this point just to make sure you have the compass set correctly (see below).
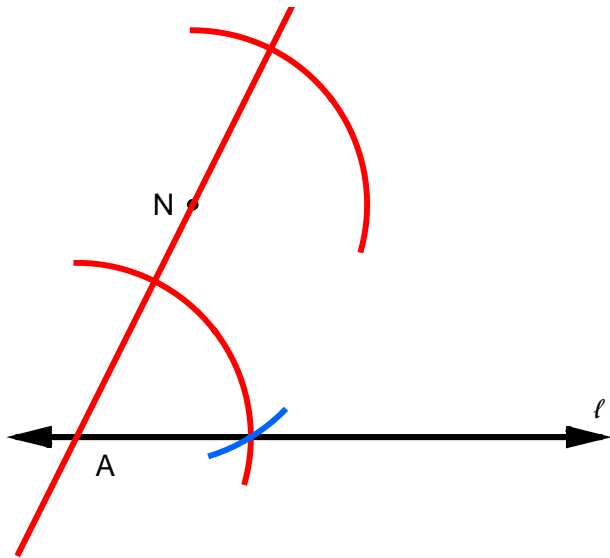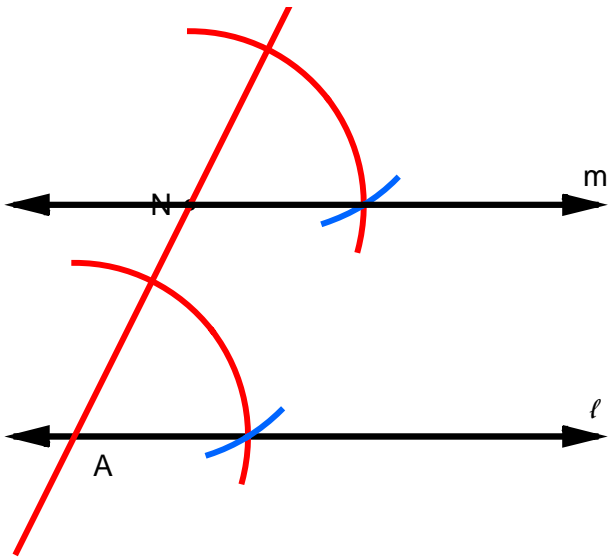
Now, transfer this same distance to the other arc by placing the compass point on the point where the second red arc (the top one) intersects $\overleftrightarrow{NA}$ and make a small arc that intersects this arc. See the figure below.
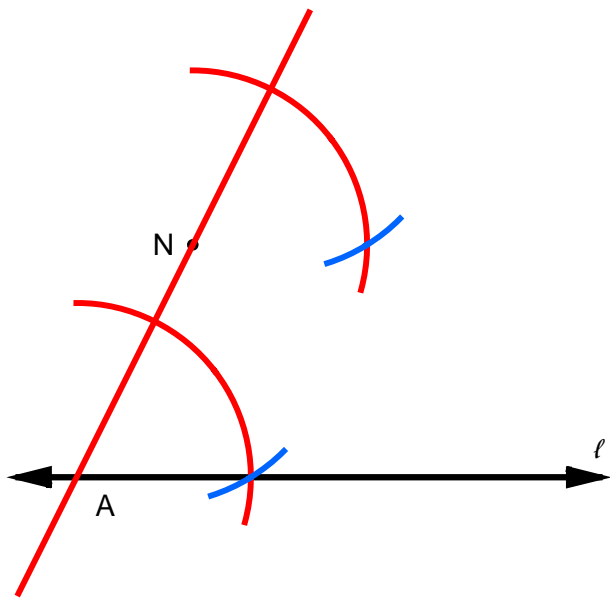
Finally, draw a line through point N and the intersection between the upper two arcs. Label this line m.

Line l is parallel to line m. The symbol for "parallel" is ‖.   The lines are parallel because they form congruent angles with the third line that intersects both of them. A line that intersects two distinct lines in a plane is called a **transversal**. The two congruent angles we constructed are known as **corresponding angles**. Corresponding angles are formed whenever two coplanar lines are cut by a transversal. However, if they are congruent then the lines must be parallel. So, in summary, what we did was construct a transversal and another line so that corresponding angles were congruent. This assured us that the two lines, l and m, would be parallel.

The next construction utilizes what we learned in construction 3 so you may want to review that construction before you move on.

**Construction 6: Perpendicular through a Point on a Line**

Using the techniques given in this construction we can construct a perpendicular to a given line through a given point on the line. We will start with a figure like the one below.

m

P
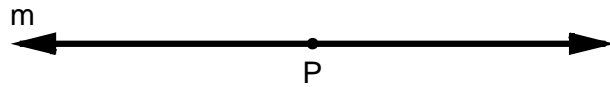
To construct a perpendicular to line *m* through point P we will "make" point P the midpoint of a segment contained in the line and construct a perpendicular bisector to the segment through point P. If the line we construct is perpendicular to this segment then it is perpendicular to the line. First we put our compass point on point P and draw arcs intersecting line *m* in two points. Label the points A and B as shown below.
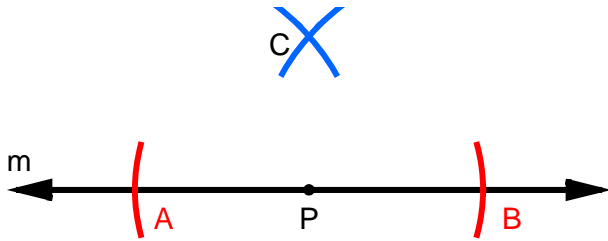
m

A   P   B
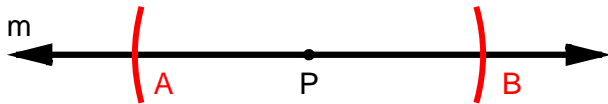
Open the compass a little wider. Place the compass point on point A and draw an arc above point P as shown below:
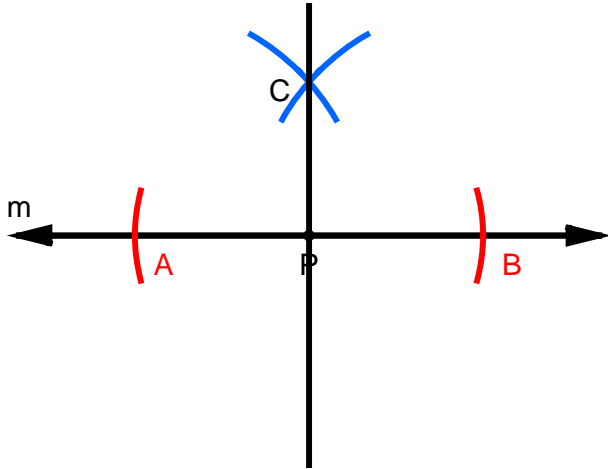
m

A   P   B
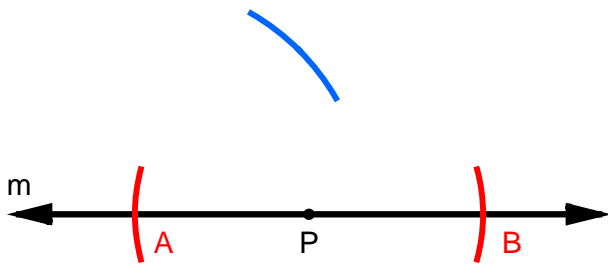
Now, with the compass setting the same make a similar arc above point P placing the compass point at point B and label the point of intersection of the two arcs C as shown below:

Finally, we draw the line through points P and C.

Since $\overleftrightarrow{PC}$ is perpendicular to $\overline{AB}$ and $\overline{AB}$ is part of line m then it follows that $\overleftrightarrow{PC}$ must be perpendicular to line m.

Construction 7: Perpendicular through a Point not on a Line.

This construction is very much like construction #6, only one step back. You will see what I mean as we get started.

We start with a figure like the one below.

P •

m
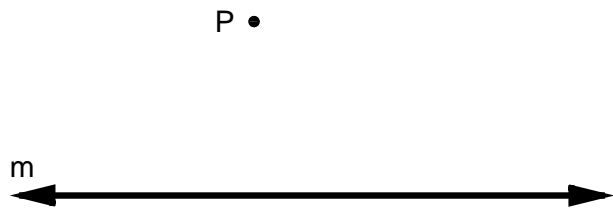
Set your compass to a length slightly larger than the distance from point P to line m. Put the compass point on point P and make two small arcs intersecting line m in two points. Label the points E and F as shown below.

P •

m
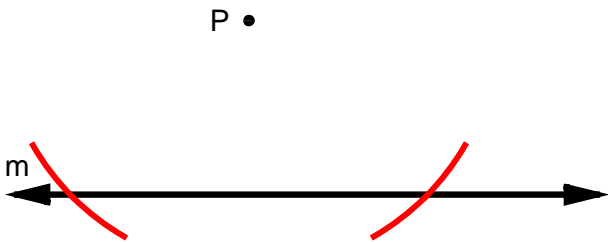
Keep your compass setting the same (you can change it if you want to but the beauty of most constructions is in their simplicity). First, place the compass point at point E and make a small arc below line m. Do the same with the compass point at point F. Label the point where these two arcs intersect G as shown below.
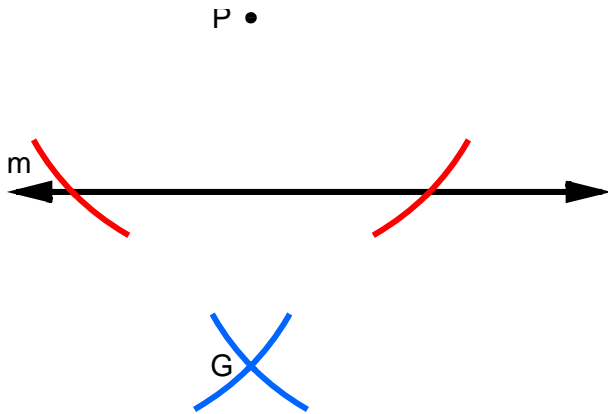
Finally, use your straightedge to construct a line through points P and G. This line is perpendicular to line m.
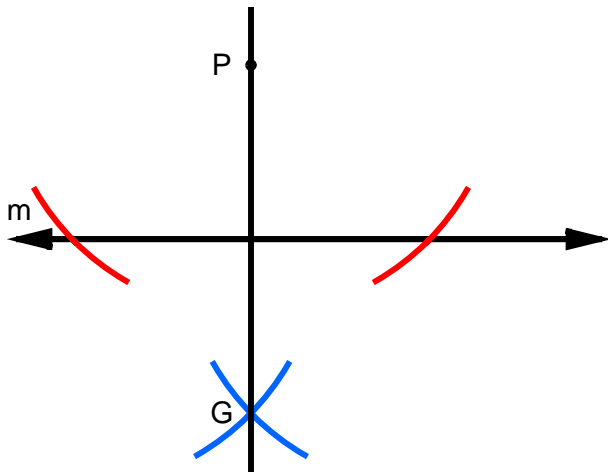
The reason this construction works (as we shall discuss in a later section) is that we constructed a segment on line m whose endpoints were equidistant from point P. Then we constructed another point (point G) that was also equidistant from points E and F. Points that are equidistant from the endpoints of a segment lie on the perpendicular bisector of the segment.

Well, that's the end of our chapter on basic constructions. There are more constructions that we can do but most of them will require you to use two or more of the basic constructions at one time. We will revisit constructions as necessary to reinforce some of the properties of polygons that will be introduced in a later chapter.