AP Calculus AB
Worksheet
Related Rates

If several variables that are functions of time t are related by an equation, we can obtain a relation involving their rates of change by differentiating with respect to t. The key to solving related rate problems is finding the equation that relates the varaibles. If the problem involves motion in paths perpendicular to each other the equation may come from the Pythagorean Theorem. If the problem involves the area or volume of a geometric figure than the appropriate area or volume formula will be involved. The following is a list of guidelines for solving related rate problems.

<u>Suggestions for Solving Related Rates Problems</u>

Step 1. Sketch a figure if helpful
Step 2. Identify all relevant variables, including those whose rates are given and those whose rates are to be found.
Step 3. Express all given rates and rates to be found as derivatives.
Step 4. Find an equation connecting the variables in step 2.
Step 5. Implicitly differentiate the equation found in step 4, using the chain rule where appropriate, and substitute in all given values.
Step 6. Solve for the derivative that will give the unknown rate.

Example: A 26 foot ladder is placed against a wall. If the top of the ladder is sliding down the wall at 2 feet per second, at what rate is the bottom of the ladder moving away from the wall when the bottom of the ladder is 10 feet from the wall.

Solution: Sketch a figure of the ladder against a wall (not shown here). Obviously the motion of the top of the ladder and the bottom of the ladder are perpendicular to each other so the Pythagorean theorem is involved.

Let's let y be the distance from the top of the ladder to the base of the wall and let x be the distance of the bottom of the ladder from the base of the wall. Then dy/dt equals the velocity of the top of the ladder and dx/dt equals the velocity of the bottom of the ladder. Our equation relating the variables is

$$x^2 + y^2 = 26^2$$

Differentiating with respect to time t gives us

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

We are given dy/dt = -2 ft./sec (the top of the ladder is sliding **down** the wall) and we are trying to find dx/dt. Solving for dx/dt gives us

$$\frac{dx}{dt} = -\frac{y}{x}\frac{dy}{dt}$$

So dx/dt depends on y, x, and dy/dt. We are given dy/dt and x but not y. However, we can find y using the Pythagorean theorem. At the instance in time when the bottom of the 26 foot ladder is 10 feet from the wall, the top of the ladder is 24 feet from the base of the wall. So

$$\frac{dx}{dt} = -\frac{24}{10}(-2) = 4.8 \text{ feet per second.}$$

Exercises.

1. If one leg AB of a right triangle increases at the rate of 2 inches per second, while the other leg AC decreases at 3 inches per second, find how fast the hypotenuse is changing when AB = 6 feet and AC = 8 feet.

2. The diameter and height of a paper cup in the shape of a cone are both 4 inches, and water is leaking out at the rate of $\frac{1}{2}$cubic inches per second. Find the rate at which the water level is dropping when the diameter of the surface is 2 inches. (Hint: use similar triangles to eliminate one of the variables in the volume formula)

3. A balloon is being filled with helium at the rate of 4 ft^3 / min. Find the rate, in square feet per minute, at which the surface area is increasing when the volume is $\frac{32\pi}{3}$ ft^3.

4. A vertical circular cylinder has radius r ft and height h ft. The height and radius both increase at the constant rate of 2 ft/sec. Find the rate, in square feet per second, at which the lateral area increases when $r = 4$ ft and $h = 10$ ft.

5. Two cars are traveling along perpendicular roads, car A at 40 mph, car B at 60 mph. At noon, when car A reaches the intersection, car B is 90 miles away and moving toward it. Find the rate at which the distance between the cars is changing at 1 P.M..