AP Calculus
Worksheet: Indefinite and Definite Integrals

1. $\int_1^2 (4x^3 - 6x)\,dx =$

(A) 2 (B) 4 (C) 6 (D) 36 (E) 42

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2. If $\int_a^b f(x)\,dx = a + 2b$, then $\int_a^b (f(x) + 5)\,dx =$

(A) a + 2b + 5 (B) 5b - 5a (C) 7b - 4a (D) 7b - 5a

(E) 7b - 6a

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3. $\frac{1}{2}\int e^{\frac{t}{2}}\,dt =$

(A) $e^{-t} + C$ (B) $e^{-\frac{t}{2}} + C$ (C) $e^{\frac{t}{2}} + C$ (D) $2e^{\frac{t}{2}} + C$ (E) $e^t + C$

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4. The area of the region enclosed by the graph of $y = x^2 + 1$ and the line $y = 5$ is

(A) $\frac{14}{3}$ (B) $\frac{16}{3}$ (C) $\frac{28}{3}$ (D) $\frac{32}{3}$ (E) 8π

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5. $\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{\cos^2 x}\,dx$ is

(A) 0 (B) 1 (C) e - 1 (D) e (E) e + 1

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6. The average value of cos x on the interval [-3, 5] is

(A) $\frac{\sin 5 - \sin 3}{8}$ (B) $\frac{\sin 5 - \sin 3}{2}$ (C) $\frac{\sin 3 - \sin 5}{2}$

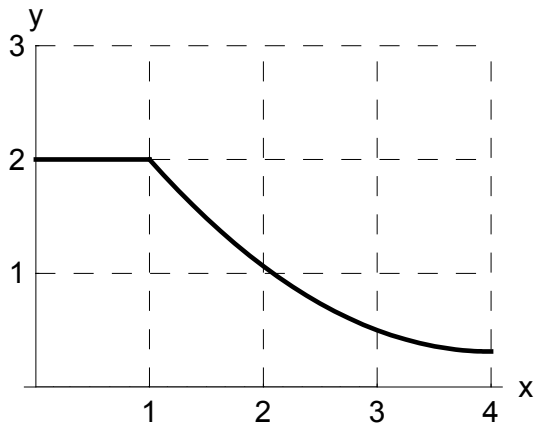
(D) $\frac{\sin 3 + \sin 5}{2}$ (E) $\frac{\sin 3 + \sin 5}{8}$

7. $\int x \sin(2x)\, dx =$

(A) $-\frac{x}{2}\cos(2x) + \frac{1}{4}\sin(2x) + C$ (B) $-\frac{x}{2}\cos(2x) - \frac{1}{4}\sin(2x) + C$

(C) $\frac{x}{2}\cos(2x) - \frac{1}{4}\sin(2x) + C$ (D) -2x cos(2x) + sin(2x) + C

(E) -2x cos(2x) - 4 sin(2x) + C



8. The graph of f is shown in the figure above. If $\int_1^3 f(x)\, dx = 2.3$and F '(x) = f(x), then F(3) - F(0) =

(A) 0.3 (B) 1.3(C) 3.3(D) 4.3 (E) 5.3

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9. What is the area of the region in the first quadrant enclosed by thegraphs of y = cos x, y = x, and the y-axis?

(A) 0.127 (B) 0.385 (C) 0.400 (D) 0.600 (E) 0.947

10. At time $t \geq 0$, the acceleration of a particle moving on the x-axis is a(t) = t + sin t. At t = 0, the velocity of the particle is -2. For what value of t will the velocity of the particle be zero?

(A) 1.02 (B) 1.48 (C) 1.85 (D) 2.81 (E) 3.14

x	0	0.5	1.0	1.5	2.0
f(x)	3	3	5	8	13

11. A table of values for a continuous function f is shown above. If four equal subintervals of [0, 2] are used, which of the following is the trapezoidal approximation of $\int_0^2 f(x)\,dx$?

(A) 8 (B) 12 (C) 16 (D) 24 (E) 32

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12. Which of the following are antiderivatives of f(x) = sin x cos x?

I. $F(x) = \frac{\sin^2 x}{2}$

II. $F(x) = \frac{\cos^2 x}{2}$

III. $F(x) = \frac{-\cos(2x)}{4}$

(A) I only (B) II only (C) III only (D) I and III only (E) II and III only

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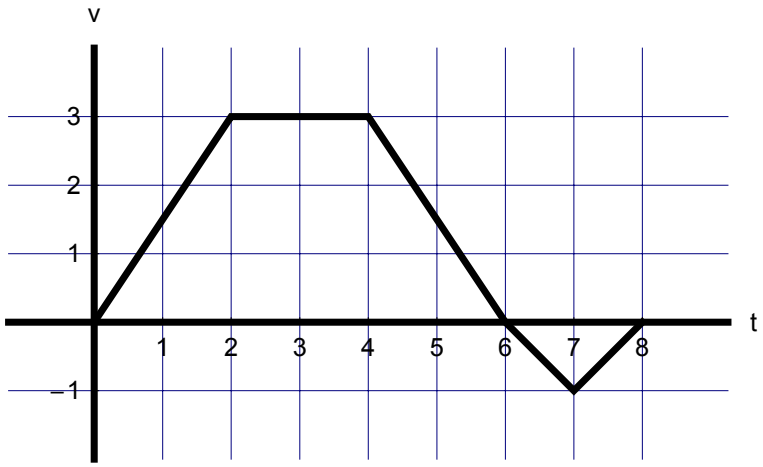

A bug begins to crawl up a vertical wire at time t = 0. The velocity of the bug at time t, $0 \le t \le 8$, is given by the function whose graph is shown above.

13. At what value of t does the bug change direction?

(A) 2 (B) 4 (C) 6 (D) 7 (E) 8

14. What is the total distance the bug traveled from t = 0 to t = 8?

(A) 14 (B) 13 (C) 11 (D) 8 (E) 6

15. What is the acceleration of the bug at t = 5?

(A) 0 (B) - 1.5 (C) 1.5(D) 3 (E) -3

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16. The area of the region enclosed by the curve y = $\frac{1}{x-1}$, the x-axis, and the lines x = 3 and x = 4 is

(A) $\frac{5}{36}$ (B) $\ln\frac{2}{3}$ (C) $\ln\frac{4}{3}$ (D) $\ln\frac{3}{2}$ (E) ln 6

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17. The acceleration of a particle moving along the x-axis at time *t* is given by *a(t)* = 6t - 2. If the velocity is 25 when *t* = 3 and the position is 10 when *t* = 1, then the position *x(t)* =

(A) $9\,t^2 + 1$
(B) $3\,t^2 - 2\,t + 4$
(C) $t^3 - t^2 + 4\,t + 6$
(D) $t^3 - t^2 + 9\,t - 20$
(E) $36\,t^3 - 4\,t^2 - 77\,t + 55$

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18. $\int \frac{3x^2}{\sqrt{x^3+1}}\,dx =$

(A) $2\sqrt{x^3+1}+C$ (B) $\frac{3}{2}\sqrt{x^3+1}+C$ (C) $\sqrt{x^3+1}+C$

(D) $\ln\sqrt{x^3+1}+C$ (E) $\ln(x^3+1)+C$

19. $\int (x^2+1)^2\, dx =$

(A) $\frac{(x^2+1)^3}{3} + C$ (B) $\frac{(x^2+1)^3}{6x} + C$ (C) $\left(\frac{x^3}{3} + x\right)^2 + C$

(D) $\frac{2x(x^2+1)^3}{3} + C$ (E) $\frac{x^5}{5} + \frac{2x^3}{3} + x + C$

20. An antiderivative for $\frac{1}{x^2-2x+2}$ is

(A) $-(x^2-2x+2)^{-2}$ (B) $\ln(x^2-2x+2)$ (C) $\ln\left|\frac{x-2}{x+1}\right|$

(D) Arcsec (x - 1) (E) Arctan (x - 1)

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21. $\int_1^{500}(13^x - 11^x)\, dx + \int_2^{500}(11^x - 13^x)\, dx =$

(A) 0.000 (B) 14.946 (C) 34.415 (D) 46.000 (E) 136.364

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22. $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{4-x^2}} =$

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{1}{2}\ln 2$ (E) -ln 2

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23. If the definite integral $\int_0^2 e^{x^2}\, dx$ is first approximated using two <u>inscribed</u> rectangles of equal width and then approximated by using the trapezoidal rule with n = 2, the difference between the two approximations is

(A) 53.60 (B) 30.51 (C) 27.80 (D) 26.80 (E) 12.78

24. If the second derivative of f is given by $f''(x) = 2x - \cos x$, which of the following could be f(x)?

(A) $\frac{x^3}{3} + \cos x - x + 1$ (B) $\frac{x^3}{3} - \cos x - x + 1$ (C) $x^3 + \cos x - x + 1$

(D) $x^2 - \sin x + 1$ (E) $x^2 + \sin x + 1$

25. $\frac{d}{dx} \int_0^x \cos(2\pi u)\, du$ is

(A) 0 (B) $\frac{1}{2\pi} \sin x$ (C) $\frac{1}{2\pi} \cos(2\pi x)$

(D) $\cos(2\pi x)$ (E) $2\pi \cos(2\pi x)$

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26. $\int x f(x)\, dx =$

(A) x f(x) - $\int x f'(x)\, dx$ (B) $\frac{x^{\square}}{2}$ f(x) - $\int \frac{x^2}{2} f'(x)\, dx$

(C) x f(x) - $\frac{x^2}{2} f(x) + C$ (D) x f(x) - $\int f'(x)\, dx$

(E) $\frac{x^2}{2} \int f(x)\, dx$

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27. $\int_0^1 x(x^2 + 2)^2\, dx =$

(A) $\frac{19}{2}$ (B) $\frac{19}{3}$ (C) $\frac{9}{2}$ (D) $\frac{19}{6}$ (E) $\frac{1}{6}$